LESSON 10

What's This When It Comes Together?

Geometry—Parts and Shapes, 3-D

Objectives . . . Students will be able to:

- *Visualize 3-D shapes.*
- *Construct 3-D shapes.*
- *Recognize relationships between 2-D and 3-D shapes.*

NCTM Standards and NAEP Strand

Concepts learned and applied in Lesson 10 correlate to NCTM Standards 1, 2, 3, 4, and 9; and NAEP Strand 3 (Geometry and Spatial Sense).

Materials List

Each Group

- 8 1/2 X 14 inch paper for recording arrangements
- five 3 X 3 inch cards (Note: these can be made from 3 X 5 inch index cards)
- tape

Each Student

- A copy of BLMs 10-A and 10-B for the "You Try It at Home" activities.

To Prepare Your Students

Introduce today's video topic and ask questions designed to ascertain if the students are familiar with the Student Prerequisites and Key Words and Terms. Present some simple examples of the type of problem to be explored—e.g., show them grid paper and ask them what it might be useful for. Ask them to describe a 3-D shape and encourage them to use the words face, edge, corner, etc.

Video Overview

A girl unloads a bag of groceries and shows us different shapes of the packages inside—a cylinder, circle, rectangle, square, etc. How are shapes important to us?

Video Lesson

Making Open-Topped Boxes

Dee Hannan of Dewey School shows her students a simple box she made out of index cards and tape. She asks her class, "How many ways can you arrange 5 squares so that, when folded, you end up with an

Student Prerequisites

- Ability to describe and make two-dimensional figures.
- Ability to recognize three-dimensional objects.

Key Words and Terms

box	*grid paper*
square	*corners, edges, and faces*

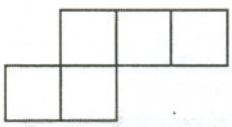
YOUR CLASS MAY WANT TO TRY THIS NOW

Break up the class into small groups and give each group some 3X3 cards, tape, and paper. After they have made an open-topped box shape, tell them that they must partially dismantle it into one flat shape and trace on the paper the outline of the shape (a pentamino). There are 12 possible arrangements using the 5 cards; however, only 6 will fold into an open-topped box.

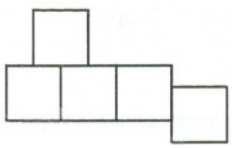
You can help your students with this problem by asking the following questions:

1. What is an arrangement? It is important to discuss the placement of the squares. Make sure that students understand that flips and rotations do not constitute a different arrangement.

   A *pentamino* is a shape made up of 5 identical square tiles that are connected along the edges.

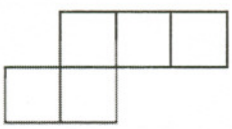


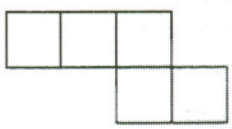
Allowed

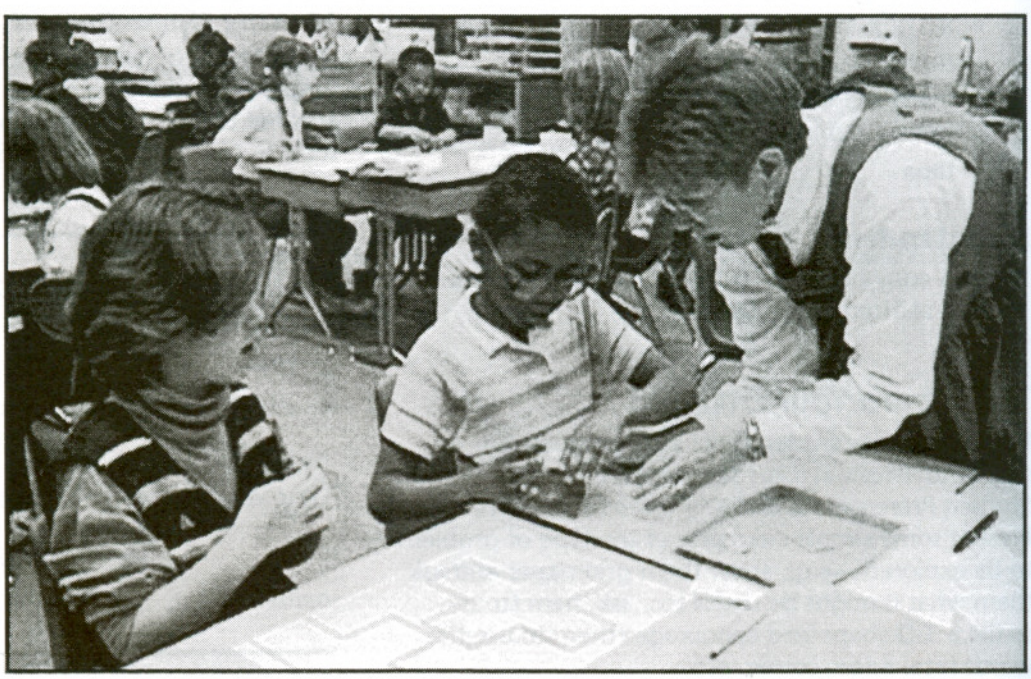


Not Allowed

Turning or flipping a pentamino does not make a different pentamino.







2. How many squares will it take to make one open-topped box? Hint: Have students try to use one piece of tape at each common edge until they are sure the arrangement will fold into a box.
3. How can you record your arrangements on the paper? How can you keep track of those which fold to make a box and those which don't?

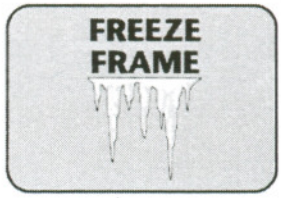
Also, you might ask the following questions to further challenge your students:

1. How many arrangements did you find? How many of those arrangements made a box?
2. Do you notice any patterns that can help you predict arrangements which will make a box?
3. Did anything surprise you when you were building the boxes?

After they have solved the problem, but before they present their solutions, play the video showing how other kids solved the problem.

Play

Watch the ways the children on the video solve the problem. The video then asks:



WOULD YOU PUT THESE SHAPES TOGETHER LIKE THIS?

Note: Question is on-screen but there is no stop sign. Teachers should pause at this point and ask each group to present its solution. Encourage your class to think about all the solutions and remind them that there are many possible solutions.

Play

Sand Box Cover

A young girl is trying to play in the sandbox, but the sand is too wet. Her older sister tells her mother that maybe she can help—if she used some plastic from the garage, maybe she could make a cover for the sandbox to keep the sand dry.

YOUR CLASS MAY WANT TO TALK ABOUT THIS NOW

Can you think of a way to make the cover?

Use the following questions to help your class solve this problem. This is a good opportunity for your students to use geometric language.

1. How would you cover the sand box?
2. What other materials could you use?
3. What could be the shape of the cover?
4. Would any other shapes work? Explain how.

Play

In the final segment, we see collections of objects such as stuffed animals, toy cars, compact discs, etc. The narrator asks if we could make boxes for each of these collections. The video suggests . . .

You Try It!

Extended Activities

These activities are designed to extend the mathematics learning introduced in this lesson.

Introduce the You Try It At Home activity (see BLM 10-A). You have just made open-top boxes with arrangements of 5 connected squares (which are also called pentaminos). Can you make several boxes with a top using 6 connected squares (hexominos)? There are eleven different possibilities.

Students' answers will vary. There are 35 different patterns that can be made with 6 squares. These are called *hexominos*. However, while there are 35 hexominos, only 11 will fold up into a cubical box. The 11 hexominos that will fold up into a closed cube are shown below:

Cross-curricular Activity

Ask your students to make sketches from the community of 3-D shapes, e.g., cylinder, prisms, a pyramid, etc. Different angles of the buildings may show different shapes. Or, using 3-D shapes make a tour map of your community. Mark important sites at which a tour bus might stop.

Writing Activity

Using one of the arrangements that formed a box in class, make a mystery word box. Choose a geometry word and write it on the bottom of the box. On the sides of the box, write clues to help other students discover the mystery word. Have your students write out the steps they went through to discover the word.

Example:

Clues: 8 corners

6 faces

12 edges

edges have equal lengths

faces have equal areas

Answer on bottom: Cube

You Try It At Home

Two handout masters, numbered 10-A and 10-B, are provided in a booklet accompanying the Teacher's Guide; reproduce as many of these as you need. Reduced examples are also provided at the end of this lesson. These activities are designed to extend and reinforce concepts taught in the lesson, and to enable parents or other caregivers to become more involved with students' mathematics learning.

Assessment and Criteria for Scoring

These criteria provide a framework for scoring various aspects of the activity and should be shared with students at the beginning of the activity. It is hoped that you will feel free to alter them to meet your needs. Use some or all of them and/or create your own.

Advanced Response: The response includes at least four accurate solutions to the problem. The sketches of the solutions are clear and complete and the explanation of the solutions is concise and well organized.

Proficient Response: The response includes three accurate solutions to the problem. The sketches of the solutions are clear and complete and the explanation of the solutions is concise and well organized.

Nearly Proficient Response: The response includes two accurate solutions to the problem. The sketches of the solutions are clear and complete and the explanation of the solutions is concise and well organized.

Minimal Response: The response includes one solution to the problem. The sketch of the solution contains a major flaw and/or the explanation of the solution is incomplete, unclear, or inaccurate.

References and Resources

Professional Resources:
National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: The Council.
Standard 1 : Mathematics as Problem Solving.
Standard 2 : Mathematics as Communication.
Standard 3 : Mathematics as Reasoning.
Standard 4 : Mathematics Connections.
Standard 9 : Geometry and Spatial Sense.
NAEP Strand 3 (Geometry and Spatial Sense).

Suggested Children's Literature

Lakshmi Hewavisenti, *Shapes and Solids*
Kate Petty & Lisa Kopper, *What's That Shape?*
Florence Sakade, *Origami: Japanese Paper Folding*
Gloria Goldreich & Esther Goldreich. *What Can She Be? An Architect*
Lydia Sharman, *The Amazing Book of Shapes*

You Try It At Home 1 and 2

Making A Cube

Ask an adult at home or other care-giver to help you with this project.

Using 3X3 cards and tape, just as in the classroom activity, construct a box that has five sides *plus* a top. Then partially dismantle the box to trace an outline of the six-sided figure you have made. Try to find as many ways to construct the shape as you can. Make diagrams of the shapes below.

Boxing Around

Ask an adult at home or other care-giver to help you with this project. You will discover the patterns companies use to manufacture boxes.

1. Choose a small to medium-sized empty box, such as individual cereal boxes, pasta boxes, milk cartons, pudding boxes, etc.

2. Locate any seams (where the box was glued together) and carefully take them apart.

3. Flatten the unfolded box and trace its shape on paper (newspaper, construction paper, etc.).

4. Make a small sketch of your pattern below.

5. What do you notice about the shapes in your drawing?